



# **Multi-scale simulation of mechanical properties** and solid solution strengthening

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Virtual Psi-k GreenALM Hands-on Tutorial 2021 – MCL Leoben (Austria)



Computational Mechanical and Materials Engineering





## Multiscale Mechanics Modelling a brief overview

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**Computational Mechanical and Materials Engineering** 



## Materials: multi-scale and complex

Challenges for *understanding* and *modelling*:

- 1. Intrinsically multi-scale
- 2. All scales important (in general)
- 3. Complexity gives *emergent* macroscopic properties





Multiscale Mechanics Modelling \ Intro – F. Maresca (U. Groningen)

## Materials: multi-scale and complex

Atomistic/Nanoscale

(Å to few nm)

Challenges for understanding and modelling:

- Intrinsically multi-scale 1.
- **All scales important (in general)** 2.

Mavrhofer et al. (2006)

**Dislocations** 

3. **Complexity** gives *emergent* macroscopic properties



### **Nano- and Microstructures**

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Interfaces

### **Multi-scale modelling**



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### Macroscale modelling





# Solid solution strengthening

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## **Solid solution strengthening in random alloys**

Motivation 1: Large strengthening observed compared with elemental metals. Why?



Solid solution strengthening – F. Maresca (U. Groningen)

Senkov, Wilks, Scott, Miracle (2011) Intermetallics 19, 698 Maresca, Curtin (2020) Acta Mater 182, 235

![](_page_6_Figure_5.jpeg)

## Solid solution strengthening in random alloys

**Motivation 2**: Immense compositional space, trial-and-error unfeasible. **Predictive model?** 

High Entropy Alloys (HEAs) [Multi Principal Element Alloys, MPEAs] [Complex Concentrated Alloys, CCAs]

Senkov, Miracle (2017) Acta Mater 122, 448

Yeh et al. (2004) Adv Eng Mater 6, 299 Solid solutions favored by config. entropy over brittle intermetallics  $\Delta S_{\rm conf} \sim R \ln N$ R gas constant, N: # of elements [Neglects formation enthalpy, other entropy contributions]

![](_page_7_Figure_5.jpeg)

 $\mathcal{O}(40)$  metals (no toxic/radioactive/rare)

 $\rightarrow \sim 10^{78}$  combinations (Cantor 2014 Entropy 16, 4749)

>> Avogadro's number!  $(6.022 \cdot 10^{23})$ 

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### **N** component alloy

 $c_n$  concentration of the *n*-th element  $(\sum_{n=1,N} c_n = 1)$ 

Dislocation exists in homogeneous, "average" alloy

![](_page_8_Figure_6.jpeg)

Average alloy effective matrix with average properties of true random alloy, e.g.:

- 1. lattice parameter
- 2. elastic constants
- 3. generalized stacking fault energy

Leyson, Hector, Curtin (2012) Acta Mater 60, 3873 Varvenne, Luque, Curtin (2016) Acta Mater 118, 164

Solid solution strengthening – F. Maresca (U. Groningen)

### \*Recent review of solute strengthening theories: Varvenne, Leyson, Ghazisaeidi, Curtin (2017) Acta Mater 124, 660

### solute n-dislocation interaction

in Embedded Atom Method (EAM) context: Varvenne, Luque, Nöhring, Curtin (2016) PRB 93, 104201

 $\Delta U_{tot}(\zeta, w)$  total energy change of dislocation segment  $\zeta$  gliding by w

![](_page_9_Figure_2.jpeg)

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$$\langle v \rangle \rangle^2 ]^{\frac{1}{2}}$$

Varvenne, Luque, Curtin (2016) Acta Mater 118, 164 10

![](_page_10_Figure_1.jpeg)

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Maresca and Curtin (2020) Acta Mater 182, 144 Maresca and Curtin (2020) Acta Mater 182, 235

![](_page_11_Figure_1.jpeg)

Total energy change w.r.t. straight dislocation:  $\Delta E^{tot} = E_L^{tot} - E_P^{tot}$ 

 $E_L^{tot}$ : contribution of joining segments (kinks, for BCC screws)

 $E_P^{tot}$ : contribution of  $\zeta$  segments lowering the energy

 $\zeta_{\rm c}$  and  $w_c$  such that  $\Delta E^{tot}/L$  is minimum!

Energy barrier and T=0K CRSS and ( $\rightarrow$  finite T, rate-dependent behaviour) determined by minimum energy structure and energy scale parameter  $\Delta \tilde{E}_{p}$ 

Solid solution strengthening – F. Maresca (U. Groningen)

### Maresca and Curtin (2020) Acta Mater 182, 144 Maresca and Curtin (2020) Acta Mater 182, 235

![](_page_11_Figure_13.jpeg)

### Varvenne, Luque, Curtin (2016) Acta Mater 118, 164 12

![](_page_12_Figure_1.jpeg)

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Maresca and Curtin (2020) Acta Mater 182, 144 13

### Strengthening Mechanisms

Cross-kink/Dipole/Jog unpinning  $\tau_{xk}$ 

advancement  $\tau_P$ 

Intermediate (unstable)

Total energy change per total length L  $E_k$  kink energy

Length w = a (Peierls valleys spacing), too costly to form longer kinks.

Minimizing w.r.t.  $\zeta$  yields the penalization  $E_L = \frac{1}{2}E_P$ . Then,  $\kappa = 0.39$  and  $\beta = 0.72$  are determined self-consistently (numerically)

Repeating unit:

$$w_c = a$$

Characteristic length scale:

 $\rightarrow \zeta_c = \left(1.083 \frac{E_k}{\Delta \tilde{E}_n}\right)^2$ 

Characteristic energy barrier in front of 2.5  $\zeta_c$  segments:  $\rightarrow \Delta E_b = 2.7 E_k$  (also computed with three-choice model)

 $2.5 \zeta_c$ 

![](_page_13_Picture_12.jpeg)

### Maresca and Curtin (2020) Acta Mater 182, 144

![](_page_14_Figure_1.jpeg)

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Maresca and Curtin (2020) Acta Mater 182, 144 15

### Strengthening mechanisms: Peierls mechanism (low T)

 $\Delta E'_{h} = 2.7 E_{k} - 2E_{k} = 0.7E_{k}$  (2 kinks are annihilated as a 2.5 $\zeta_{c}$  segment moves forward)

![](_page_15_Figure_3.jpeg)

Strengthening mechanisms: Kink glide mechanism (intermediate T)

![](_page_16_Figure_2.jpeg)

**Total enthalpy:** 

$$H_{\rm k}(z) = 1.37 \sqrt{w_{\rm k}/b} \Delta \tilde{E}_p \ln\left(2.3 \frac{z}{w_{\rm k}} + 1\right) + 1.37 E_{\rm k} \frac{z}{2.5}$$

**Total enthalpy barrier:** 

$$\longrightarrow \Delta H_{\rm k}(\tilde{\tau}_{\rm k}) = 1.37 \sqrt{w_{\rm k}/b} \Delta \tilde{E}_p \left(\tilde{\tau}_{\rm k} - {\rm lm}\right)$$

 $ilde{ au}_{
m k}$ 

### **Stress vs enthalpy:**

$$\frac{\tau - \tau_b}{\tau_{k,0} - \tau_b} = \left[ \exp\left(0.89\Delta \tilde{H}_k + 0.5\Delta \tilde{H}_k^{\frac{1}{4}} + 0.6\right) - 1 \right]^{-1}$$

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$$\frac{\tau abz}{\zeta_c} - \frac{\tau abz}{2}$$
$$= \frac{\tau - \tau_b}{\tau_{k,0} - \tau_b}$$

### Strengthening mechanisms: Cross-kink mechanism (moderate to high T)

Standard analysis for creation of point defects:

![](_page_17_Figure_3.jpeg)

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![](_page_17_Picture_6.jpeg)

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**Screw** dislocations strengthening mechanisms: analytical theory

$$\tau(\dot{\varepsilon}, T) = \tau_{\mathrm{xk}} \left( \dot{\varepsilon}, T \right) + \min \left[ \tau_{\mathrm{k}} \left( \dot{\varepsilon}, T \right), \tau_{P} \left( \dot{\varepsilon}, T \right) \right]$$

where

$$\begin{aligned} \tau_{\rm xk} \left( \dot{\varepsilon}, T \right) &= \tau_{\rm xk,0} \left[ 1 - \left( \frac{\Delta H}{E_{\rm v/i}} \right)^{\frac{2}{3}} \right] \\ \tau_{\rm k} \left( \dot{\varepsilon}, T \right) &= \tau_b + \frac{\tau_{\rm k,0} - \tau_b}{\left\{ \exp \left[ 0.89 \frac{\Delta H}{\Delta E_{\rm k,0}} + 0.5 \left( \frac{\Delta H}{\Delta E_{\rm k,0}} \right)^{\frac{1}{4}} + 0.6 \right] - 1 \right\}}, \quad \frac{\tau - \tau_b}{\tau_{\rm k,0} - \tau_b} > \frac{1}{3.45 \frac{\zeta_c}{w_{\rm k}} + 1}, \\ \tau_{\rm k} \left( \dot{\varepsilon}, T \right) &= \tau_b - (\tau_{\rm k,0} - \tau_b) \frac{w_{\rm k}}{3.45 \zeta_c} \frac{\Delta H}{\Delta E_{\rm k,0}}, \quad \frac{\tau - \tau_b}{\tau_{\rm k,0} - \tau_b} < \frac{1}{3.45 \frac{\zeta_c}{w_{\rm k}} + 1}, \\ \tau_P (\dot{\varepsilon}, T) &= \tau_{P,0} \left[ 1 - \left( \frac{\Delta H}{\Delta E_{b,P}} \right)^{\frac{2}{3}} \right]. \end{aligned}$$

Finite temperature/strain rate behaviour by using an Arrhenius model,  $\Delta H = kT$ 

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![](_page_18_Picture_8.jpeg)

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### sm

$$\ln\left(\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}}\right)$$

Maresca and Curtin (2020) Acta Mater 182, 144 19

### All theory quantities expressed w.r.t. few material parameters:

- Kink energy  $E_k$ 1)
- 2) Solute-dislocation interaction parameter  $\Delta \tilde{E}_{p}$
- 3) Vacancy/self-interstitial formation energies  $E_{v/i}$
- Peierls barrier  $\Delta V_P$ 4)

$$\zeta_c = \left(1.083 \frac{E_k}{\Delta \tilde{E}_p}\right)^2 b$$
  

$$\tau_{\rm xk,0} = \frac{\pi E_{\rm v/i}}{ab\zeta_{\rm v/i}} \qquad (\zeta_{\rm v} = 7.5\zeta_c,$$
  

$$\tau_b = \frac{1.08E_k}{ab\zeta_c}$$
  

$$\tau_{\rm k,0} = \frac{6.3\Delta \tilde{E}_p}{ab^2\sqrt{w_{\rm k}/b}} + \tau_b$$
  

$$\Delta E_{\rm k,0} = 1.37\sqrt{w_{\rm k}/b}\Delta \tilde{E}_p$$
  

$$\tau_{P,0} = \frac{\pi\Delta V_P}{ba} + \frac{0.44E_k}{ba\zeta_c} \left[1 - \frac{1}{(0,0)}\right]^2$$
  

$$\Delta E_{b,P} = \frac{(10\Delta V_P\zeta_c + 0.7E_k)^3}{(20\Delta V_P\zeta_c + 0.7E_k)^2}.$$

![](_page_19_Picture_9.jpeg)

![](_page_19_Figure_10.jpeg)

Maresca and Curtin (2020) Acta Mater 182, 144

![](_page_20_Figure_1.jpeg)

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![](_page_20_Picture_4.jpeg)

### X-kink spacing in Fe 9%Si:

### (expts Caillard: 143 +/-54 nm)

### Only 2 quantities fitted.

![](_page_21_Figure_1.jpeg)

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### Only 2 quantities fitted.

![](_page_22_Figure_1.jpeg)

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23

### Only 2 quantities fitted.

### **Conclusions – Screw theory**

**Fully derived** parameter-free theory for screw dislocation strengthenig from dilute to "high entropy" alloys.

- **Minimum energy screw dislocation kinked** in alloys: kink nucleation *not* the controlling mechanism
- The theory is based on three **main mechanisms** (Peierls, kink glide and cross-kinking) acting at different 2) temperature regimes, with **cross-kinking** dominant at higher T.
- Theory **rationalizes** the main experimental results on Fe-Si, Nb-Mo, Nb-W alloys and on Nb-Ti-Zr HEAs 3)
- In screw-controlled Nb-Ti-Zr, **no high strength retention** above 1300K.

![](_page_23_Figure_7.jpeg)

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### **Motivations:**

1. Screw-controlled Ti-Nb-Zr not retaining strength above 1300K:

different mechanism responsible for high-T strength in MoNbTaVW?

- Experiments in NbMo\* and FeSi\*\* 2. reveal decreased edge mobility w.r.t. screws
- 3. X-ray line analysis in TiZrHfNbTa indicate edge dislocations dominance at yielding\*\*\*
- 4. Yield strengths correlate with solute misfit volume (Yao et al. 2017)  $\rightarrow$  hallmark of edge strengthening!

\*Statham, Koss and Christian (1972); \*\*Caillard (2013); \*\*\*Dirras et al. (2015)

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![](_page_24_Figure_9.jpeg)

Maresca and Curtin (2020) Acta Mater 182, 235 25

### **NbTaTiV BCC High Entropy Alloy**

![](_page_25_Figure_2.jpeg)

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![](_page_25_Picture_4.jpeg)

### **CrMoNbV BCC High Entropy Alloy**

### Lee, Maresca et al. (2021) Nature Communications 12, 5474

$$\Delta E^{tot}(\zeta, w) / L = \begin{bmatrix} \kappa \frac{\Gamma w^2}{2\zeta} - \beta \left(\frac{\zeta}{2\sqrt{2}b}\right)^{\frac{1}{2}} \Delta \tilde{E}_p(w) \end{bmatrix} \frac{1}{(1+\kappa)\zeta}$$
 Total energy change per total length  

$$\bigcap_{E_L} \qquad \bigcap_{E_P} \qquad \Gamma = \frac{1}{12} \mu b^2 \text{ line tension}$$

Minimizing w.r.t.  $\zeta$  yields the penalization  $E_L = \frac{1}{4}E_P$  (lower than screw case!). Then,  $\kappa = 0.56$  and  $\beta = 0.83$  are determined self-consistently (numerically)

![](_page_26_Figure_3.jpeg)

$$\Rightarrow \Delta E_b = 1.11 \left(\frac{w_c^2 \Gamma \Delta \tilde{E}_p^2(w_c)}{b}\right)^{\frac{1}{3}} \quad \text{(from stochastic simulations)}$$

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![](_page_26_Picture_7.jpeg)

### Maresca and Curtin (2020) Acta Mater 182, 235

**One single strengthening mechanism!** 

**Advancement of**  $\zeta_c$  segments over the energy barrier  $\Delta E_b$ 

# **Total energy barrier:** $\rightarrow \Delta E(\tau) = \Delta E_b \left(1 - \frac{\tau}{\tau_{u0}}\right)^{\frac{3}{2}}$

### **T=0K flow-stress**

$$\tau_{y0} = \frac{\pi}{2} \frac{\Delta E_b}{b\zeta_c(w_c)w_c} = 1.01 \left(\frac{\Delta \tilde{E}_p^4(w_c)}{\Gamma b^5 w_c^5}\right)^{\frac{1}{3}}$$

### **Finite T, finite strain-rate flow stress**

**1) Low T** 
$$(\tau/\tau_{y0} > 0.5)$$
  
 $\tau_y(T, \dot{\varepsilon}) = \tau_{y0} \left[ 1 - \left( \frac{kT}{\Delta E_b} \ln \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}} \right)^{\frac{2}{3}} \right]$ 
**2) High T**  $(\tau/\tau_{y0} < 0.5)$   
 $\tau_y(T, \dot{\varepsilon}) = \tau_{y0} \exp \left( -\frac{1}{0.55} \frac{kT}{\Delta E_b} \ln \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}} \right)^{\frac{2}{3}} \right]$ 

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![](_page_27_Picture_10.jpeg)

[Leyson, Curtin (2016) MSMSE 24, 065005]

![](_page_27_Picture_12.jpeg)

Maresca and Curtin (2020) Acta Mater 182, 235

![](_page_28_Figure_1.jpeg)

Theory vs T=300K expts and MD simulations

<b>Theory length-scales</b>
vs MD simulations

Mo-Nb-Ta-V-W	$w_c$ theory (Å)	$w_c$ simulations (Å)	$\lambda/4$ theory (Å)	$\lambda/4$ simulations (Å)
0.0-33.3-33.3-33.3-0.0	12.0	10.0	38.7	40.2
21.7-20.6-15.6-21-21.1	12.0	8.4	48.9	49.4
25.6-22.7-24.4-0.0-27.3	12.1	9.1	61.1	72.5
24.9-25.8-26.6-22.7-0.0	12.1	11.0	45.6	52.4
0.0-28.5-29.65-20.67-21.18	12.1	9.3	48.4	50.7

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![](_page_28_Picture_7.jpeg)

Maresca and Curtin (2020) Acta Mater 182, 235

![](_page_29_Figure_1.jpeg)

![](_page_29_Picture_4.jpeg)

Maresca and Curtin (2020) Acta Mater 182, 235 30

Optimization performed over 10M compositions in AI-Cr-Mo-Nb-Ta-W-V-Ti-Zr-Hf

See:

Lee, Maresca et al. (2021) Nature Communications 12, 5474

MATLAB implementation available at

https://doi.org/10.24435/materialscloud:fs-27

![](_page_30_Figure_6.jpeg)

Taylor factor: 3.067

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### Shown here: Mo-Nb-Ta-V-W subset

![](_page_31_Figure_1.jpeg)

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## Solid solution strengthening in BCC random alloys

**Fully derived** parameter-free theory for **screw and edge** strengthenig from dilute to "high entropy" alloys.

- Minimum energy **screw dislocation is kinked** in alloys: 1) x-kink strengthening controls high-T strength (up to Tm/2).
- Edge strengthening rationalizes observed high-T strength 2) in refractory BCC HEAs: xk-unpinning not operating above Tm/2.
- Edge theory reduced to **simple form**, function of elastic moduli and volume misfits. 3)
- Theory used for **optimization** to find **new compositions** in immense BCC refractory alloys space. 4)

**Challenge:** (Single-)phase predictions *ab-initio* [see Lee, Maresca et al. 2021; Ferrari, Lysogorskyi, Drautz (2021) PRM 5, 063606]

More details in:

Maresca & Curtin (2020) Mechanistic origin of high strength in refractory BCC high entropy alloys up to 1900K. Acta Mater. 182:235 Maresca & Curtin (2020) Theory of screw dislocation strengthening in random BCC alloys from dilute to "High-Entropy" alloys. Acta Mater. 182:144 Yin, Maresca & Curtin (2020) Vanadium is an optimal element for strengthening in both fcc and bcc high-entropy alloys. Acta Mater. 188:486 Ghafarollahi, Maresca & Curtin (2019) Solute/screw dislocation interaction energy parameter for strengthening in bcc dilute to high entropy alloys. Modelling Simul. Mater. Sci. Eng. 27:085011 Lee, Maresca, Feng, Chou, Ungar, Widom, An, Poplawski, Chou, Liaw, Curtin (2021)

Strength can be controlled by edge dislocations in refractory high-entropy alloys. Nature Communications 12:5474

Solid solution strengthening – F. Maresca (U. Groningen)

![](_page_32_Picture_11.jpeg)